United College of Engineering and Research, Allahabad

Department of Computer Science & Engineering

**Question Bank**

**Discrete Structure and Theory of Logic (KCS-303)**

**Unit-1**

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| **Q. No.** | **Question** | **CO** | **Bloom’s level** |
| 1 | Define various types of functions. | CO1 | L1 |
| 2 | How many symmetric and reflexive relations are possible from a set A containing ‘n’ elements? | CO1 | L3 |
| 3 | Prove that +++……………..+>for n ≥ 2 using principle of mathematical induction | CO1 | L3 |
| 4 | Find the numbers between 1 to 500 that are not divisible by any of the integers 2 or 3 or 5 or 7. | CO1 | L4 |
| 5 | Is the “divides” relation on the set of positive integers transitive? What is the reflexive and symmetric closure of the relation?  R = {(a, b) | a > b} on the set of positive integers? | CO1 | L3 |
| 6 | Determine the power set P(A) of A = {a, b, c, d}. | CO1 | L2 |
| 7 | Define surjective function. | CO1 | L1 |
| 8 | Let f and g be the functions from the set of integers to the set of integers defined by f (x) = 2x + 3 and g(x) = 3x + 2. What is the composition of f and g? What is the composition of g and f? | CO1 | L2 |
| 9 | Consider the following relations on set {1, 2, 3, 4}:  R1 = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)},  R2 = {(1, 1), (1, 2), (2, 1)},  R3 = {(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4)},  R4 = {(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3)},  R5 = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 3), (3, 4), (4, 4)},  R6 = {(3, 4)}.  Which of these relations are reflexive? | CO1 | L2 |
| 10 | List all the ordered pairs in the relationR = {(a, b) | a divides b} on the set {1, 2, 3, 4,5, 6} and also display the graphical representation of the same. | CO1 | L2 |
| 11 | Prove the proposition P(n) that the sum of the first n positive integers is ; that is, P(n) = 1 + 2 + 3+・・・+n = | CO1 | L3 |
| 12 | Determine whether each of these statements is true or false.  i) 0 ∈∅ ii) ∅∈ {0} iii) {0} ⊂∅ iv) ∅⊂ {0}  v) {0} ∈ {0} vi ){0} ⊂ {0} vii) {∅} ⊆ {∅} | CO1 | L2 |
| 13 | For each of these relations on the set {1, 2, 3, 4}, decide whether it is reflexive, whether it is symmetric, whether it is anti-symmetric, and whether it is transitive   1. {(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)} 2. {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)} 3. {(2, 4), (4, 2)} 4. {(1, 2), (2, 3), (3, 4)} 5. {(1, 1), (2, 2), (3, 3), (4, 4)} 6. {(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)} | CO1 | L4 |
| 14 | Let A and B be sets. Show that AXB≠ BXA. Under what condition AXB=BXA? | CO1 | L2 |
| 15 | Let R be a binary relation on the set of all positive integers such that:  R= { (a, b) | a-b is an odd positive integers}  Is R reflexive ? Symmetric? Transitive? | CO1 | L2 |
| 16 | Define Multiset and Power set. Determine the power set A={1,2}. | CO1 | L2 |
| 17 | Let f: X→Y and X=Y=R, the set of real number. Find f-1if   1. fr(x)=x2 2. f(x)=(2x-1)/5 | CO1 | L2 |
| 18 | Prove by using mathematical induction that:  7+77+777+............+777...........7=7/81[10n+1-9n-10] for every n ϵ N. | CO1 | L4 |
| 19 | Let R be a relation on R, the set of real numbers, such that  R={(x,y)│׀x-y׀<1}. Is R an equivalence relation? Justify. | CO1 | L3 |
| 20 | Let R be a relation on the set of natural numbers N, as R = {(x, y): x, y ∈ N, 3x + y = 19}. Find the domain and range of R. Verify whether R is reflexive. | CO1 | L2 |
| 21 | Show that the relation R on the set Z of integers given by R = {(a, b): divides a – b}, is an equivalence relation. | CO1 | L3 |
| 22 | Proof by induction: +++……………..+ = | CO1 | L4 |

**Unit-2**

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| **Q. No.** | **Question** | **CO** | **Bloom’s level** |
| 1 | Let Z be the group of integers with binary operation \* defined by  a\*b = a + b − 2, for all a, b∈ Z . Find the identity element of the group (Z,\*). | CO2 | L3 |
| 2 | Justify that “If a,b are the arbitrary elements of a group G then (ab)2 = a2b2 if and only if G is abelian. | CO2 | L2 |
| 3 | What do you mean by cosets of a subgroup? Consider the group Z of integers under addition and the subgroup H = {…., -12, -6, 0, 6 12, ……} considering of multiple of 6   1. Find the cosets of H in Z 2. What is the index of H in Z. | CO2 | L3 |
| 4 | What is Ring? Define elementary properties of Ring with example. | CO2 | L2 |
| 5 | Prove or disprove that intersection of two normal subgroups of a groupG is again a normal subgroup of G. | CO2 | L3 |
| 6 | Consider the group G = {1, 2, 3, 4, 5, 6} under multiplication modulo 7.  (i) Find the multiplication table of G.  (ii) Find 2−1, 3−1, 6−1.  (iii) Find the orders and subgroups generated by 2 and 3.  (iv) Is G cyclic? | CO2 | L3 |
| 7 | Define the Subgroup of a group. | CO2 | L1 |
| 8 | Explain Cyclic group. Let H be a subgroup of a finite group G. Justify the statement “the order of H is a divisor of the order of G”. | CO2 | L3 |
| 9 | Prove that (R,+,\*) is a ring with zero divisors, where R is 2x2 matrix and + and \* are usual addition and multiplication operations. | CO2 | L3 |
| 10 | Let (A,\*) be a monoid such that for every x in A , x\*x=e, where e is the identity element.Show that (A,\*) is an abelian group. | CO2 | L3 |
| 11 | Define ring and give an example of a ring with zero divisors. | CO2 | L2 |
| 12 | State and prove Lagrange’s theorem for group. | CO2 | L3 |
| 13 | Prove that every cyclic group is an abelian group. | CO2 | L4 |
| 14 | Obtain all distinct left cosets of { 0, 3 } in the group ( Z6, +6 ) and find their union. | CO2 | L2 |
| 15 | Let (G,\*) be a group, where \* is usual multiplication operation on G. Show that for any a, b ϵ G,   1. (a-1)-1 = a 2. (ab)-1 = b-1a-1 | CO2 | L3 |
| 16 | Prove that the set S={0,1,2,3} forms a ring under addition and multiplication modulo 4 but not a field. | CO2 | L3 |
| 17 | Define the following with suitable example: (i) Cyclic group (ii) Zero divisor of ring. | CO2 | L2 |
| 18 | Let G={1,-1,i,-i} be the multiplicative group, where i=   1. Determine whether G is an abelian. 2. If G is a cyclic group, then determine generator of G. | CO2 | L4 |
| 19 | Prove that intersection of two subgroups is also a subgroup. | CO2 | L3 |
| 20 | Show that every group of order 3 is cyclic group. | CO2 | L2 |

**Unit-3**

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| **Q. No.** | **Question** | **CO** | **Bloom’s level** |
| 1 | Prove that a lattice with 5 elements is not a Boolean algebra. | CO3 | L3 |
| 2 | Show that the following are equivalent in a Boolean algebra  a ≤ b⇔ a\*b' = 0⇔b' ≤ a’⇔ a’⊕ b = 1 | CO3 | L3 |
| 3 | Let (L,∨,∧,≤) be a distributive lattice and a, b∈ L . if a ∧ b = a ∧ c and  a ∨ b = a ∨ c then show that b = c | CO3 | L3 |
| 4 | Find the values of the Boolean function represented by  F (x, y, z) = xy + z’. | CO3 | L3 |
| 5 | Prerequisites in college is a familiar partial ordering of available classes. We write A ≺B if course A is a prerequisite for course B. Let C be the ordered set consisting of the mathematics courses and their prerequisites appearing in figure.    i. Draw the Hasse diagram for the partialordering C of these classes.  ii. Find all minimal and maximal elements of C.  iii. Does C have a first element or a last element? | CO3 | L4 |
| 6 | Answer these questions for the poset({3, 5, 9, 15,24, 45}, |).  i. Find the maximal elements. ii. Find the minimal elements.  iii. Is there a greatest element? iv. Is there a least element?  v. Find all upper bounds of {3, 5}.vi. Find the least upper bound of {3, 5}.  vii. Find all lower bounds of {15, 45}. viii.Find the greatest lower bound of {15, 45},if it exists. | CO3 | L4 |
| 7 | Which of the partially ordered sets in Fig are lattices? | CO3 | L2 |
| 8 | Define a Partial Ordering . | CO3 | L1 |
| 9 | Describe the Boolean duality principle. Write the dual of each Boolean equations:   1. x+x’y=x+y 2. (x.1)(0+x’)=0. | CO3 | L2 |
| 10 | Draw the Haase diagram of [p(a,b,c),≤], Find greatest element , least element ,minimal element & maximal element. | CO3 | L3 |
| 11 | Simplify the following Boolean function using three variables maps:   1. f(x,y,z)=Σ(0,1,5,7) 2. f(x,y,z)=Σ(1,2,3,6,7) | CO3 | L3 |
| 12 | Show that the “greater than or equal” relation (>=) is a partial ordering on the set of integers. | CO3 | L3 |
| 13 | Distinguish between bounded lattice and complemented lattice. | CO3 | L2 |
| 14 | In a Lattice if a≤b≤c , then show that   1. a∨b=b∧c 2. (a∨b)∨(b∧c) = (a∨b)∧ (a∨c) = b | CO3 | L3 |
| 15 | Prove that every finite subset of a lattice has an LUB and a GLB. | CO3 | L3 |
| 16 | Give an example of a lattice which is a modular butnot a distributive. | CO3 | L2 |
| 17 | Find the Boolean algebra expression for thefollowing system. | CO3 | L2 |
| 18 | Define a Boolean function of degree n. Simplify the following Boolean expression using K-map:  xyz + xy’z +x’y’z +x’yz +x’y’z’ | CO3 | L3 |
| 19 | Draw Hasse diagram on divisibility relation on the following set  A = {3, 4, 12, 24, 48, 72} | CO3 | L2 |
| 20 | If the lattice is represented by the Hasse diagram given below     1. Find all the complements of ‘e’. 2. Prove that this lattice is bounded complemented lattice. | CO3 | L3 |

**Unit-4**

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| **Q. No.** | **Question** | **CO** | **Bloom’s level** |
| 1 | Write the contra positive of the implication: “if it is Sunday then it is a holiday”. | CO4 | L2 |
| 2 | Show that the propositions 𝑝→𝑞 𝑎𝑛𝑑 ¬𝑝∨𝑞 are logically equivalent. | CO4 | L2 |
| 3 | Show that ((P ∨Q) ∧¬(¬ Q∨¬ R)) ∨ ( ¬ P∨¬ Q) ∨ ( ¬ P∨¬ R) is a tautology by using equivalences. | CO4 | L3 |
| 4 | Obtain the principle disjunctive and conjunctive normal forms of the formula ( p→r) ∧ ( q↔ p) | CO4 | L3 |
| 5 | Explain various Rules of Inference for Propositional Logic. | CO4 | L1 |
| 6 | Prove the validity of the following argument “if the races are fixed so the casinos are crooked, then the tourist trade will decline. If the tourist trade decreases, then the police will be happy. The police force is never happy.Therefore, the races are not fixed. | CO4 | L3 |
| 7 | Verify that the given propositions are tautology or not.   1. p ∨￢ (p ∧q) 2. ￢p ∧q | CO4 | L2 |
| 8 | Prove that (P˅Q)→(P˄Q) is logically equivalent to P↔Q. | CO4 | L3 |
| 9 | Express this statement using quantifiers:  “Every student in this class has taken some course in every department in the school ofmathematical sciences”. | CO4 | L3 |
| 10 | Constructed the truth table for the following statements:   1. (P→Q’)→P’ 2. P↔(P’∨Q’). | CO4 | L3 |
| 11 | Show the implications without constructing the truth table  (P🡪Q) 🡪Q ⇒ P∨Q. | CO4 | L3 |
| 12 | Write the symbolic form and negate the following statements :   * Everyone who is healthy can do all kinds of work. * Some people are not admired by everyone. * Everyone should help his neighbors, or his neighborswill not help him. | CO4 | L2 |
| 13 | Differentiate between tautology and contradiction with suitable examples. | CO4 | L2 |
| 14 | Show that statements P🡪Q and Q’ 🡪 P’ are logically equivalent. | CO4 | L2 |
| 15 | Prove the validity of the following argument:-  If I get the job and work hard then I will get promoted. If I will get promoted, then I will be happy. I will not be happy. Therefore I will not get the job or I will not work hard. | CO4 | L3 |
| 16 | The contra-positive of a statement S is given as “ If x<2 then x+4 < 6” . Write the statement S and its converse. | CO4 | L2 |
| 17 | Show that ((P∨Q)∧ ~(~P∧ (~Q∨~R)))∨ (~P∧ ~R)∨ (~P ∨R) is tautology without using truth table. | CO4 | L4 |
| 18 | Rewrite the following arguments using quantifiers, variables and predicate symbols.   1. All birds can fly. 2. Some men are genius. 3. Some numbers are not rational. 4. There is a student who likes mathematics but not geography. | CO4 | L3 |
| 19 | “If the labour market is perfect then the wages of all persons in a particular employment will be equal. But it always the case that wages for such persons are not equal therefore the labour market is not perfect”. Test the validity of this argument using truth table. | CO4 | L3 |
| 20 | Express the following statements using quantifiers and Iogical connectives.   1. Mathematics book that is published in India has ablue cover. 2. AII animals are mortal. All human being are animal.Therefore, ail human being are mortar. 3. There exists a mathematics book with a cover thatis not blue. 4. He eats crackers only ifhe drinks milk. 5. There are mathematics books that are publishedoutside India. 6. Not all books have bibliographies. | CO4 | L3 |

**Unit-5**

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| **Q. No.** | **Question** | **CO** | **Bloom’s level** |
| 1 | Show that there does not exist a graph with 5 vertices with degrees 1, 3,4, 2, 3 respectively. | CO5 | L2 |
| 2 | Obtain the generating function for the sequence 4, 4, 4, 4, 4, 4, 4. | CO5 | L3 |
| 3 | Define Pigeon-hole principle. | CO5 | L1 |
| 4 | Define planar graph. Prove that for any connected planar graph, v – e+ r = 2 Where v, e, r is the number of vertices, edges, and regions of the graph respectively. | CO5 | L2 |
| 5 | Solve the following recurrence equation using generating function  G (K) -7 G (K-1) + 10 G (K-2) = 8K + 6 | CO5 | L4 |
| 6 | A collection of 10 electric bulbs contain 3 defective ones  (i) In how many ways can a sample of four bulbs be selected?  (ii) In how many ways can a sample of 4 bulbs be selected which contain  2 good bulbs and 2 defective ones?  (iii) In how many ways can a sample of 4 bulbs be selected so that either  the sample contains 3 good ones and 1 defectives ones or 1 good and 3defectives ones? | CO5 | L4 |
| 7 | Draw all trees with exactly six vertices. | CO5 | L2 |
| 8 | Find the adjacency matrix A = [aij] of graph given in following figure:- | CO5 | L2 |
| 9 | What are the degrees and what are the neighborhoods of the vertices in the graphs G and H displayed in Figure ? | CO5 | L2 |
| 10 | For which values of n do these graphs have an Euler path but no Euler circuit?  i. Kn ii.Cn iii. Wn iv.Qn | CO5 | L3 |
| 11 | Are the graphs G and H displayed in the figure bipartite? | CO5 | L3 |
| 12 | Represent the expressions (x + xy) + (x/y) and x + ((xy + x)/y) using binary trees. Write these expressions in:   1. Prefix notation. 2. Postfix notation. 3. Infix notation. | CO5 | L3 |
| 13 | Construct the ordered rooted tree whose preorder traversal is a, b, f, c, g, h, i, d, e, j, k, l, where a has four children, c has three children, j has two children, b and e have one child each, and all other vertices are leaves. | CO5 | L4 |
| 14 | What are the chromatic numbers of the graphs G and H shown in figure | CO5 | L2 |
| 15 | How many cards must be selected from a standard deck of 52 cards to guarantee that at least three cards of the same suit are chosen? | CO5 | L3 |
| 16 | What is a binary Search tree? Explain with example. | CO5 | L2 |
| 17 | Determine the value of each of their prefix expressions:   1. -\*2/933 2. +-\*335/↑232 | CO5 | L2 |
| 18 | Define preorder, inorder and postorder tree traversal. Give an example of preorder, postorder&inorder traversal of a binary tree of your choice with at least 12 vertices. | CO5 | L4 |
| 19 | Solve the recurrence relation by the method of generating function.  ar-7ar-1+10ar-2=0,r≥2, Given a0=3 and a1=3. | CO5 | L4 |
| 20 | Find the recurrence relation from yn = A2n + B(–3)n. | CO5 | L3 |
| 21 | State the applications of binary search tree. | CO5 | L2 |
| 22 | Define Multigraph. Explain with example in brief. | CO5 | L2 |
| 23 | Let G be a graph with 10 vertices. If 4 vertices has degree 4and 6 vertices has degree 5, then find the number of edges of G. | CO5 | L3 |
| 24 | Prove that a simple graph with n vertices and k components can have at most edges. | CO5 | L3 |
| 25 | Solve the recurrence relation yn+2 – 5yn+1 + 6yn = 5n  subject to the condition y0 = 0, y1 = 2. | CO5 | L3 |
| 26 | Prove that a connected graph G is Euler graph if and only if every vertex of G is of even degree. | CO5 | L4 |
| 27 | Which of the following simple graph have a Hamiltonian circuit or, if not a Hamiltonian path? | CO5 | L2 |
| 28 | Solve the recurrence relation using generating function:  an-7an-1+10an-2 = 0 with a0=3 , and a1=3. | CO5 | L4 |
| 29 | Solve the recurrence relation  ar+2-5ar+1+6ar = (r+1)2 | CO5 | L3 |
| 30 | Explain the following terms with examples.   1. Homomorphism and Isomorphism graphs 2. Euler and Hamiltonian Graph 3. Planar and Complete bipartite graph | CO5 | L2 |